Lecturered By. Dr. H. K. Yadav.
Deptt. of Mathematics
Marwari College, Darbhanga.
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Complex Numbers and Quadratic Eqns.

Miscellaneous Exercise.

Q.17. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$

Soln. $\because$ $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\frac{(\beta - \alpha)\bar{\beta}}{(1 - \bar{\alpha}\beta)\bar{\beta}}\right| = \left|\frac{(\beta - \alpha)\bar{\beta}}{\bar{\beta} - \bar{\alpha}\beta\bar{\beta}}\right|$

$= \left|\frac{(\beta - \alpha)\bar{\beta}}{\bar{\beta} - \bar{\alpha}}\right|$ $[\because \beta\bar{\beta} = |\beta|^2 = |1|^2 = 1]$

$= \frac{|(\beta - \alpha)\bar{\beta}|}{|\bar{\beta} - \bar{\alpha}|}$ $\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$= \frac{|\beta - \alpha||\bar{\beta}|}{|\overline{\beta - \alpha}|}$ $[\because |z_1 z_2| = |z_1||z_2|$ and $|\bar{z}_1 - \bar{z}_2| = |\overline{z_1 - z_2}|]$

$= \frac{\cancel{|\beta - \alpha|}\,|\beta|}{\cancel{|\beta - \alpha|}}$ $[\because |\bar{z}| = |z|]$

$= |\beta| = 1.$

Q.18. Find the number of non-zero integral solutions of the eqn $|1 - i|^x = 2^x$.

Soln. $\because |1 - i|^x = 2^x \Rightarrow \sqrt{1^2 + (-1)^2}^{\,x} = 2^x \Rightarrow \sqrt{(1+1)}^{\,x} = 2^x$

$\Rightarrow \sqrt{2}^{\,x} = 2^x \Rightarrow 2^{\frac{x}{2}} = 2^x \Rightarrow \frac{x}{2} = x \Rightarrow 2x - x = 0$
$\Rightarrow x = 0.$

But we need the non-zero solutions.
Hence, the number of solutions is zero.

Q.20. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

Soln: $\because \left(\frac{1+i}{1-i}\right)^m = 1$

$$\Rightarrow \left[\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right]^m = 1$$

$$\Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^m = 1$$

$$\Rightarrow \left[\frac{1+2i+i^2}{1+1}\right]^m = 1 \quad \left[\because (1+i)^2 = 1^2+2i+i^2 \text{ and } i^2=-1\right]$$

$$= \cancel{1}+2i-\cancel{1} \text{ and } i^2=-1$$
$$= 2i$$

$$\Rightarrow \left[\frac{\cancel{1}+2i-\cancel{1}}{2}\right]^m = 1$$

$$\Rightarrow \left[\frac{\cancel{2}i}{\cancel{2}}\right]^m = 1$$

$$\Rightarrow i^m = 1$$

$$\Rightarrow (\sqrt{-1})^m = 1 \quad \left[\because i^2=-1 \Rightarrow i=\sqrt{-1}\right]$$

$$\Rightarrow (-1)^{\frac{m}{2}} = 1$$

$$\Rightarrow (-1)^{\frac{m}{2}} = (-1)^2$$

$$\Rightarrow \frac{m}{2} = 2$$

$$\Rightarrow m = 4$$

Hence, the least positive integral value of m is 4

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